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Exercise 1.4.3

a. We must prove that: $\frac{a_0}{a_n} = \frac{p\lambda}{q\beta}$

$$a_n\left(\frac{p}{q}\right)^n + \dots + a_0 = 0$$

let $\dots = \gamma$

$$a_n\left(\frac{p}{q}\right)^n + \gamma + a_0 = 0$$

$$a_n\left(\frac{p}{q}\right)^n + \gamma = a_0$$

$$\left(\frac{p}{q}\right)^n + \frac{\gamma}{a_n} = \frac{a_0}{a_n}$$

$$\frac{p}{q}\underbrace{\left(\left(\frac{p}{q}\right)^{n-1} + \frac{\gamma}{a_n}\left(\frac{p}{q}\right)^{-1}\right)}_{\text{this must be an element of } \mathbb{Q}} = \frac{a_0}{a_n}$$

then $\frac{p}{q}\left(\frac{\lambda}{\beta}\right) = \frac{a_0}{a_n}$

(2)

b. If $a_n = 1$

then $\frac{a_0}{a_n} = a_0$

then $\left(\frac{p}{q}\right)\left(\frac{\lambda}{\beta}\right)$ must be $\in \mathbb{Z}$,

because $a_0 \in \mathbb{Z}$

for $q\beta = 1$, $q, \beta = 1$ because

$q, \beta \in \mathbb{Z}$

then $\left(\frac{p}{q}\right)$ is $\in \mathbb{Z}$

c. consider $f(x) = x^2 - 2$

roots:

$0 = x^2 - 2$

$x \cdot x = 2$

since 2 is a perfect sauare, no integer exists that can satisfy the eauation. so

x cannot be rational

③ consider $f(x) = x^3 - \sqrt{12}$

12 is a perfect cube, same applies